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FUZZY STABILITY FOR A CLASS OF QUADRATIC FUNCTIONAL EQUATIONS

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Abstract. In this paper, we investigate the following form of a certain class of quadratic functional equations and its fuzzy stability.

$$f(kx+y)+f(kx-y)=f(x+y)+f(x-y)-2(1-k^2)f(x)$$

where k is a fixed rational number with $k \neq 1, -1, 0$.

1. Introduction and preliminaries

In 1964, S. M. Ulam proposed the following stability problem(cf [13]) : "Let G_1 be a group and G_2 a metric group with the metric d. Given a constant $\delta > 0$, does there exist a constant $c > 0$ such that if a mapping $f : G_1 \longrightarrow G_2$ satisfies $d(f(xy), f(x)f(y)) < c$ for all $x, y \in G_1$, then there exists a unique homomorphism $h : G_1 \longrightarrow G_2$ with $d(f(x), h(x)) < \delta$ for all $x \in G_1$?"

In 1941, D. H. Hyers [5] answered this problem under the assumption that the groups G_1 and G_2 are Banach spaces. Aoki [1] and Rassias [12] generalized the Hyers' result. Rassias [12] solved the generalized Hyers-Ulam stability of the functional inequality

$$\|f(x+y)-f(x)-f(y)\| \leq \epsilon(\|x\|^p+\|y\|^p)$$

for some $\epsilon \geq 0$ and p with $p < 1$ and all $x, y \in X$, where $f : X \longrightarrow Y$ is a function between Banach spaces. The paper of Rassias[12] has provided a lot of influence in the development of what we call the generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability of functional equations. A generalization of the Rassias theorem was obtained by

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Gavruta [4] by replacing the unbounded Cauchy difference by a general control function in the sprit of Rassias approach. Kim, Han and Shim [8] investigated Hyers-Ulam stability for a class of quadratic functional equations via a typical form

$$f(ax+by)+f(ax-by)-2a^2f(x)-2b^2f(y)$$
$$+c[f(x+y)+f(x-y)-2f(x)-2f(y)]=0.$$

In this paper, we consider the fuzzy version of stability for the class of quadratic fuctional equations

$$f(kx+y)+f(kx-y)=f(x+y)+f(x-y)-2(1-k^2)f(x)$$

in the fuzzy normed space setting. The concept of fuzzy norm on a linear space was introduced by Katsaras [7] in 1984. Bag and Samanta [2], following Cheng and Mordeson [3], gave an idea of a fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [9]. In 2008, A. K. Mirmostafaee and M. S. Moslehian [10,11] used the definition of a fuzzy norm in [2] to obtain a fuzzy version of stability for the additive functional equation

$$f(x+y)=f(x)+f(y), \tag{1.1}$$

and the quadratic functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y). \tag{1.2}$$

We call a solution of (1.1) *an additive mapping* or briefly *additive* and a solution of (1.2) is called *a quadratic mapping* or briefly *quadratic*.

Now, we introduce the following functional equation for fixed rational number k with $k \neq 1, -1, 0$:

$$f(kx+y)+f(kx-y)=f(x+y)+f(x-y)-2(1-k^2)f(x) \tag{1.3}$$

in a fuzzy normed space. It is easy to see that the function $f(x)=px^2$ is a solution of the functional equation (1.3). Now, we recall the following definition for a fuzzy normed space given in [12] and the fundamental concepts:

Definition 1.1. Let X be a real linear space. A function $N : X \times \mathbb{R} \longrightarrow [0,1]$ is called *a fuzzy norm on X* if for all $x, y \in X$ and all $s, t \in \mathbb{R}$,

(N1) $N(x,\ t)=0$ for $t \leq 0$;
(N2) $x=0$ if and only if $N(x,\ t)=1$ for all $t>0$;
(N3) $N(cx,\ t)=N(x,\frac{t}{|c|})$ if $c \neq 0$;
(N4) $N(x+y,\ s+t) \geq min\{N(x,s),\ N(y,t)\}$;

(N5) $N(x,\cdot)$ is a nondecreasing function of $\mathbb{R}$ and $\lim_{t\longrightarrow\infty} N(x,\ t) = 1$;
(N6) for $x \neq 0$, $N(x,\ t)$ is continuous on $\mathbb{R}$.
In this case, the pair (X, N) is called *a fuzzy normed linear space.*

The examples of fuzzy norms and properties of fuzzy normed linear spaces are given in [10,11].

Definition 1.2. Let (X, N) be a fuzzy normed linear space. A sequence $\{x_n\}$ in X is said to be *convergent* if there exists an $x \in X$ such that $\lim_{n\longrightarrow\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called *the limit of the sequence* $\{x_n\}$ and we denote it by $N - \lim_{n\longrightarrow\infty} x_n = x$.

Definition 1.3. Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to be *Cauchy* if for each $\epsilon > 0$ and each $t > 0$ there is an $m \in N$ such that for all $n \geq m$ and all $p > 0$, we have $N(x_{n+p} - x_n, t) > 1 - \epsilon$.

It is well known that every convergent sequence in a fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be *complete* and the fuzzy complete normed space is called *a fuzzy Banach space.*

2. Solution of (1.3)

In this section, we investigate solutions of (1.3). In Theorem 2.2, we conclude that any solution of (1.3) is quadratic if k is a rational number with $k \neq 1, -1, 0$. We start with the following lemma.

Lemma 2.1. *If $f : X \longrightarrow Y$ satisfies (1.3) for $k \neq 0$, then the following equation holds.*

$$\begin{aligned}(-1+\frac{1}{k^2})&[f(2x+y)+f(2x-y)]\\ &-2(-1+\frac{1}{k^2})[f(x+y)+f(x-y)]\\ &+(-1+\frac{1}{k^2})f(y)+(-1+\frac{1}{k^2})f(-y)\\ &+4(-1+\frac{1}{k^2})f(x)-2(-1+\frac{1}{k^2})f(2x)=0\end{aligned} \tag{2.1}$$

for all $x, y \in X$.

Proof. Putting $x = 0 = y$ in (1.3), we get $f(0) = 0$. Letting $y = 0$ in (1.3), we have

$$f(kx) = k^2 f(x) \tag{2.2}$$

for all $x \in X$.

Replacing y by $x + y$ in (1.3), we have

$$\begin{aligned} &f((k+1)x+y)) + f((k-1)x-y)) - [f(2x+y) + f(-y)] \\ &= 2(k^2-1)f(x) \end{aligned} \tag{2.3}$$

for all $x, y \in X$ and letting $y = -y$ in (2.3), we have

$$\begin{aligned} &f((k+1)x-y)) + f((k-1)x+y)) - [f(2x-y) + f(y)] \\ &= 2(k^2-1)f(x) \end{aligned} \tag{2.4}$$

for all $x, y \in X$.

Replacing x and y by $x + \frac{1}{k}y$ and x in (1.3) respectively, we have

$$\begin{aligned} &f((k+1)x+y)) + f((k-1)x+y)) - [f(2x + \frac{1}{k}y) + f(\frac{1}{k}y)] \\ &= 2(k^2-1)f(x + \frac{1}{k}y) \end{aligned} \tag{2.5}$$

for all $x, y \in X$ and letting $y = -y$ in (2.5), we have

$$\begin{aligned} &f((k+1)x-y)) + f((k-1)x-y)) - [f(2x - \frac{1}{k}y) + f(-\frac{1}{k}y)] \\ &= 2(k^2-1)f(x - \frac{1}{k}y) \end{aligned} \tag{2.6}$$

for all $x, y \in X$. By (2.3), (2.4), (2.5), and (2.6), we have

$$\begin{aligned} &-[f(2x+y) + f(2x-y) + f(y) + f(-y)] \\ &+ \frac{1}{k^2}[f(y) + f(-y)] + \frac{1}{k^2}[f(2ax+y) + f(2ax-y)] \\ &= 4(k^2-1)f(x) - 2(1 - \frac{1}{k^2})[f(ax+y) + f(ax-y)] \end{aligned} \tag{2.7}$$

for all $x, y \in X$. By (1.3) and (2.7), we have

$$\begin{aligned} &-[f(2x+y) + f(2x-y) + f(y) + f(-y)] + \frac{1}{k^2}[f(y) + f(-y)] \\ &+ \frac{1}{k^2}[f(2x+y) + f(2x-y) - 2f(2x) - 2f(y) - 2k^2 f(2x) - 2f(y)] \\ &= 4(k^2-1)f(x) \\ &- 2(1 - \frac{1}{k^2})[f(x+y) + f(x-y) - 2f(x) - 2f(y) - 2k^2 f(x) - 2f(y)] \end{aligned}$$

for all $x, y \in X$. Now, just simplifying this equation, we can get the result. $\square$

THEOREM 2.2. *Suppose that $f : X \longrightarrow Y$ satisfies (1.3). Then f is quadratic.*

Proof. Suppose that f satisfies (1.3). Then by (2.1) in lemma 2.1, we have

$$\frac{1}{k^2}(1-k^2)[f(2x+y)+f(2x-y)]$$
$$= \frac{1}{k^2}(1-k^2)[2f(x+y)+2f(x-y)-4f(x)+2f(2x)-f(y)-f(-y)]$$

for all $x, y \in X$. Hence by [6], f is quadratic-cubic. Now since $f(kx) = k^2 f(x)$, f is quadratic. □

3. Fuzzy stability for the functional equation (1.3)

Let X be a real linear space, (Y, N) be a fuzzy Banach space and (Z, N') be a fuzzy normed space, respectively. As a matter of convenience, for a given mapping $f : X \longrightarrow Y$, we use the abbreviation

$$Df(x,y) = f(kx+y) + f(kx-y) - [f(x+y)+f(x-y)-2(1-k^2)f(x)]$$

for all $x, y \in X$. Now we will prove fuzzy version of stability for the functional equation (1.3).

THEOREM 3.1. *Let $f : X \longrightarrow Y$ be a mapping such that $f(0) = 0$ and*

$$N(Df(x,y),\ t) \geq N'(\phi(x,y),\ t) \tag{3.1}$$

for all $x, y \in X$ and all $t > 0$. Let $\phi : X^2 \longrightarrow Z$ be a function and r be a real number such that $0 < |r| < k^2$ such that

$$N'(\phi(kx,ky),\ t) \geq N'(r\phi(x,y),\ t) \tag{3.2}$$

for all $x, y \in X$ and all $t > 0$. Then there exists a unique quadratic mapping $Q : X \longrightarrow Y$ such that the inequality

$$N(Q(x)-f(x),\ t) \geq N'(\frac{1}{2(k^2-|r|)}\phi(x,0),\ t) \tag{3.3}$$

holds for all $x \in X$ and all $t > 0$.

Proof. Inequality (3.1) is equivalent to the following :

(3.4)
$$N(f(kx+y)+f(kx-y)-[f(x+y)+f(x-y)+2(1+k^2)f(x)],\ t)$$
$$\geq N'(\phi(x,y),\ t)$$

for all $x, y \in X$ and all $t > 0$. By (3.2) and (N3), we have

$$N'(\phi(k^n x, k^n y),\ t) \geq N'(r^n \phi(x,y),\ t) = N'\Big(\phi(x,y),\ \frac{t}{|r|^n}\Big) \tag{3.5}$$

for all $x, y \in X$ and all $t > 0$ and so by (3.5), we have

$$N'(\phi(k^n x, k^n y),\ |r|^n t) \geq N'(\phi(x,y),\ t) \tag{3.6}$$

for all $x, y \in X$ and all $t > 0$. Letting $y = 0$ in (3.4), by (N3), we have

$$N\Big(\frac{f(kx)}{k^2} - f(x),\ \frac{t}{2k^2}\Big) \geq N'(\phi(x,0),\ t) \tag{3.7}$$

for all $x \in X$ and all $t > 0$. By (3.2), (3.6), (3.7), and (N3), we have

$$N\Big(\frac{f(k^{n+1}x)}{k^{2(n+1)}} - \frac{f(k^n x)}{k^{2n}},\ \frac{|r|^n t}{2k^{2(n+1)}}\Big) \geq N'(\phi(k^n x, 0),\ |r|^n t) \geq N'(\phi(x,0),\ t) \tag{3.8}$$

for all $x \in X$, all $t > 0$ and all positive integers n. Hence by (3.8) and (N4), for any $x \in X$, we have

$$\begin{aligned}
&N\Big(\frac{f(k^n x)}{k^{2n}} - f(x),\ \sum_{i=0}^{n-1} \frac{|r|^i t}{2k^{2(i+1)}}\Big) \\
&= N\Big(\sum_{i=0}^{n-1} \Big[\frac{f(k^{i+1}x)}{k^{2(i+1)}} - \frac{f(k^i x)}{k^{2i}}\Big],\ \sum_{i=0}^{n-1} \frac{|r|^i t}{2k^{2(i+1)}}\Big) \\
&\geq \min\Big\{N\Big(\frac{f(k^{i+1}x)}{k^{2(i+1)}} - \frac{f(k^i x)}{k^{2i}},\ \frac{|r|^i t}{2k^{2(i+1)}}\Big) \mid 0 \leq i \leq n-1\Big\} \\
&\geq N'(\phi(x,0),\ t)
\end{aligned} \tag{3.9}$$

for all $x \in X$, all $t > 0$ and all positive integers n. So for any $x \in X$, we have

$$\begin{aligned}
&N\Big(\frac{f(k^{m+p}x)}{k^{2(m+p)}} - \frac{f(k^m x)}{k^{2m}},\ \sum_{i=m}^{m+p-1} \frac{|r|^i t}{2k^{2(i+1)}}\Big) \\
&= N\Big(\sum_{i=m}^{m+p-1} \Big[\frac{f(k^{i+1}x)}{k^{2(i+1)}} - \frac{f(k^i x)}{k^{2i}}\Big],\ \sum_{i=m}^{m+p-1} \frac{|r|^i t}{2k^{2(i+1)}}\Big) \\
&\geq \min\Big\{N\Big(\frac{f(k^{i+1}x)}{k^{2(i+1)}} - \frac{f(k^i x)}{k^{2i}},\ \frac{|r|^i t}{2k^{2(i+1)}}\Big) \mid m \leq i \leq m+p-1\Big\} \\
&\geq N'(\phi(x,0),\ t)
\end{aligned} \tag{3.10}$$

for all $x \in X$, all $t > 0$ and all positive integers m, p. Thus, by (3.10) and (N3), for any $x \in X$, we have

$$N\Big(\frac{f(k^{m+p}x)}{k^{2(m+p)}} - \frac{f(k^m x)}{k^{2m}},\ t\Big) \geq N'\Big(\phi(x,0),\ \frac{t}{\sum_{i=m}^{m+p-1} \frac{|r|^i}{2k^{2(i+1)}}}\Big) \tag{3.11}$$

for all $x \in X$, all $t > 0$ and all positive integers m and p. Since $\sum_{i=0}^{\infty} \frac{|r|^i}{2k^{2(i+1)}}$ is convergent, $\lim_{m\longrightarrow\infty} \frac{t}{\sum_{i=m}^{m+p-1} \frac{|r|^i}{2k^{2(i+1)}}} = \infty$ and so $\Big\{\frac{f(k^m x)}{k^{2m}}\Big\}$ is a Cauchy sequence in (Y, N). Since (Y, N) is a fuzzy Banach space, there is a mapping $Q : X \longrightarrow Y$ defined by

$$\begin{aligned} Q(x) &= N - \lim_{n\longrightarrow\infty} \frac{f(k^n x)}{k^{2n}} \quad \textit{or} \\ \lim_{n\longrightarrow\infty} N\Big(Q(x) &- \frac{f(k^n x)}{k^{2n}},\ t\Big) = 1,\ \ t > 0 \end{aligned} \tag{3.12}$$

for all $x \in X$. Moreover by (3.9), we have

$$N\Big(\frac{f(k^n x)}{k^{2n}} - f(x),\ t\Big) \geq N'\Big(\phi(x,0),\ \frac{t}{\sum_{i=0}^{n-1} \frac{|r|^i}{2k^{2(i+1)}}}\Big) \tag{3.13}$$

for all $x \in X$, all $t > 0$ and all positive integers m, p. Let ϵ be a real number $0 < \epsilon < 1$. Then, by (3.12), (3.13), and (N4), we have

$$\begin{aligned} &N(Q(x) - f(x),\ t) \\ &\geq \min\Big\{N\Big(Q(x) - \frac{f(k^n x)}{k^{2n}},\ \epsilon t\Big), N\Big(\frac{f(k^n x)}{k^{2n}} - f(x),\ (1 - \epsilon t)\Big)\Big\} \\ &\geq N'\Big(\phi(x,0),\ \frac{(1-\epsilon)t}{\sum_{i=0}^{n-1} \frac{|r|^i}{2k^{2(i+1)}}}\Big) \\ &\geq N'(\phi(x,0),\ 2(1-\epsilon)(k^2 - |r|)t) \end{aligned} \tag{3.14}$$

for sufficiently large positive integer n, all $x \in X$, and all $t > 0$. Since $N(x, \cdot)$ is continuous on $\mathbb{R}$, we get

$$N(Q(x) - f(x),\ t) \geq N'(\phi(x,0),\ 2(k^2 - |r|)t) \tag{3.15}$$

for all $x \in X$ and all $t > 0$ and so we have (3.3).

By (3.2) and (N5), we have

$$
(3.16)\qquad
\begin{aligned}
&N\Big(\frac{Df(k^n x, k^n y)}{k^{2n}},\ t\Big)\\
&\geq N'(\phi(k^n x, k^n y),\ k^{2n} t)\\
&\geq N'\Big(\phi(x,y),\ \frac{k^{2n}}{|r|^n} t\Big)
\end{aligned}
$$

for all $x, y \in X$ and all $t > 0$. Since $\lim_{n\longrightarrow\infty} N'\Big(\phi(x,y),\ \frac{k^{2n}}{|r|^n}t\Big) = 1$, by (3.12), (3.16), and (N4), we have

(3.17)

$$
\begin{aligned}
&N(DQ(x,y),\ t)\\
&\geq \min\Big\{N\Big(DQ(x,y) - \frac{Df(k^n x, k^n y)}{k^{2n}},\ \frac{t}{2}\Big),\ N\Big(\frac{Df(k^n x, k^n y)}{k^{2n}},\ \frac{t}{2}\Big)\Big\}\\
&\geq N\Big(\frac{Df(k^n x, k^n y)}{k^{2n}},\ \frac{t}{2}\Big)\\
&\geq N'\Big(\phi(x,y),\ \frac{k^{2n}}{2|r|^n} t\Big),\ t > 0
\end{aligned}
$$

for sufficiently large n, all $x, y \in X$ and all $t > 0$, because

$$
\lim_{n\longrightarrow\infty} N\Big(Q(x,y) - \frac{Df(k^n x, k^n y)}{k^{2n}},\ t\Big) = 1
$$

for all $x \in X$ and all $t > 0$. Since $\lim_{n\longrightarrow\infty} N'\Big(\phi(x,y),\ \frac{k^{2n}}{|r|^n}t\Big) = 1$, $N(DQ(x,y),\ t) = 1$ for all $t > 0$ and so, by (N2), $DQ(x,y) = 0$ for all $x, y \in X$. By Theorem 2.2, Q is quadratic.

To prove the uniquness of Q, let $Q_1 : X \longrightarrow Y$ be another quadratic mapping satisfying (3.3). Then for any $x \in X$ and a positive integer n, $Q_1(k^n x) = k^{2n} Q_1(x)$ and so by (3.13),

(3.18)

$$
\begin{aligned}
&N(Q(x) - Q_1(x),\ t)\\
&\geq \min\Big\{N\Big(\frac{Q(k^n x)}{k^{2n}} - \frac{f(k^n x)}{k^{2n}},\ \frac{t}{2}\Big), N\Big(\frac{Q_1(k^n x)}{k^{2n}} - \frac{f(k^n x)}{k^{2n}},\ \frac{t}{2}\Big)\Big\}\\
&\geq N'(\phi(k^n x, 0),\ k^{2n}(k^2 - |r|)t)\\
&\geq N'\Big(\phi(x,0),\ \frac{k^{2n}(k^2 - |r|)t}{|r|^n}\Big)
\end{aligned}
$$

holds for all $x \in X$, all positive integer n, and all $t > 0$. Since $|r| < k^2$, $\lim_{n \longrightarrow \infty} N'\Big(\phi(x,0), \ \frac{k^{2n}(k^2-|r|)t}{|r|^n}\Big) = 1$ and so $Q(x) = Q_1(x)$ for all $x \in X$. $\square$

Similar to Theorem 3.1, we have the following theorem :

THEOREM 3.2. *Let $f : X \longrightarrow Y$ be a mapping such that $f(0) = 0$ and*

$$N(Df(x,y), \ t) \geq N'(\phi(x,y), \ t) \tag{3.19}$$

for all $x, y \in X$ and all $t > 0$. Let $\phi : X^2 \longrightarrow Z$ be a function and r be a real number such that $0 < k^2 < |r|$ such that

$$N'\Big(\phi\Big(\frac{x}{k}, \frac{y}{k}\Big), \ t\Big) \geq N'\Big(\frac{1}{r}\phi(x,y), \ t\Big) \tag{3.20}$$

for all $x, y \in X$ and all $t > 0$. Then there exists a unique quadratic mapping $Q : X \longrightarrow Y$ such that the inequality

$$N(Q(x) - f(x), \ t) \geq N'(\frac{1}{2(|r| - k^2)}\phi(x,0), \ t) \tag{3.21}$$

holds for all $x \in X$ and all $t > 0$.

We can use Theorem 3.1 and Theorem 3.2 to get a classical result in the framework of normed spaces.

For any normed space $(X, || \cdot ||)$, the mapping $N_X : X \times \mathbb{R} \longrightarrow [0,1]$, defined by

$$N_X(x,t) = \begin{cases} 0, & \text{if } t \leq 0 \\ \frac{t}{t+||x||}, & \text{if } t > 0 \end{cases}$$

a fuzzy norm on X. Using this, we have the following corollary :

COROLLARY 3.3. *Let $f : X \longrightarrow Y$ be a mapping such that $f(0) = 0$ and*

$$\|Df(x,y)\| \leq \|x\|^p \|y\|^p + \|x\|^{2p} + \|y\|^{2p}. \tag{3.22}$$

for all $x, y \in X$, a fixed rational number k and a fixed real number p such that $|k| > 1$ and $0 < p < 1$ or $|k| < 1$ and $p > 1$. Then there exists a unique quadratic mapping $Q : X \longrightarrow Y$ such that the inequality

$$\|Q(x) - f(x)\| \leq \frac{\|x\|^{2p}}{|k^2 - k^{2p}|}$$

holds for all $x \in X$.

References

[1] T. Aoki, *On the stability of the linear transformation in Banach space*, J. Math. Soc. Japan **2** (1950), 64-66.

[2] T. Bag and S. K. Samanta, *Finite dimensional fuzzy normed linear spaces*, J. Fuzzy Math. **3** (2003), 687-705.

[3] S. C. Cheng and S. N. Mordeson, *Fuzzy linear operator and Fuzzy normed linear spaces* , Bull. Calcutta Math. Soc. **86** (1994), 429-436.

[4] P. Gavruta, *A generalization of the Hyers-Ulam-Rassias of approximatel additive mappings* , J. Math. Anal. **184** (1994), 431-436.

[5] D. H. Hyers, *On the stability of the linear functional equations* , Proc. Nat. Acad. Sci. USA **27** (1941), 222-224.

[6] K. W. Jun, H. M. Kim, and I. S. Chang, *On the Hyers-Ulam stability of an Euler-Lagrange type cubic functional equation*, J. Comp. Anal. and Appl. **7** (2005), 21-33.

[7] A. K. Katsaras, *Fuzzy topological vector space II Fuzzy Sets System* , Fuzzy Sets System. **12** (1984), 143-154.

[8] C. I. Kim, G. J. Han, and S. A. Shim, *Hyers-Ulam stability for a class of quadratic functional equations via a typical form*, Abs. Appl. Anal. **2013** (2013), 1-8.

[9] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica. **11** (1975), 326-334.

[10] A. K. Mirmostafaee and M. S. Moslehian, *Fuzzy almost quadratic functions*, Results Math. **52** (2008), 161-177.

[11] A. K. Mirmostafaee and M. S. Moslehian, *Fuzzy version of Hyers-Ulam-Rassias theorem*, Fuzzy Sets Syst. **159** (2008), 720-729.

[12] T. M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297-300.

[13] S. M. Ulam, *A Collection of Mathematial Problems*, Interscience Tracts in Pure and Applied Mathematics,no.8, Interscience, New York, NY, USA, 1964.

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